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Problem, problem-posing, and -solving: *Perspectives and cases*

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Abstract

The first aim of this study was to investigate perceptions about the concept of 'a problem'. The second aim was to then investigate posed and solved problems. This was a qualitative study conducted with 46 pre-service middle school mathematics teachers who studied at a state university in Turkiye. The 'Problem, Problem-Posing and -Solving Form' was a data collection instrument designed for this study. The data related to the concept of 'a problem' were evaluated by content analysis. The posed problems were analyzed by using an evaluation path. A rubric was used to evaluate the solved problems. The results showed that the concept of 'a problem' was perceived as a question that the individual does not like the moment s/he encounters it, requires analysis and reasoning, and needs to be solved. The posed problems were mathematical and solvable. Problems related to data processing and probability learning domains were not posed. Problems of easy and medium difficulty were generally posed at the fifth-class level. Nearly all (93%) of problems were routine. Consequently, the pre-service teachers posed routine mathematical problems that could be solved. Likewise, there were deficiencies in these problems, such as the use of mathematical terminology and the inability to create problem texts. In problem-solving, more than half of the pre-service teachers did not focus on understanding the problem, no strategy was chosen, and only a partial logical evaluation of solving a problem was made. In summary, result-oriented solutions were widely evident without considering the problem-solving stages.

Keywords: Problem, problem-posing, problem-solving, perspectives, cases

About the Article

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Introduction

What is the problem? When we encounter such a question, we think of the mathematics lessons and the problems solved in these lessons, as well as the problems we encounter in daily life. As a matter of fact, this situation arises from the impression we have at school, that the problems come from the textbooks or teachers, and that the students' task is to solve them (Brown & Walter, 2005). In this case, we first encounter the process of establishing a problem and then solving it. Understanding mathematics is also all about doing it, and problem-solving (PS) and problem-posing (PP) are at the center of mathematics (Polya, 2002) and are seen as important cognitive activities for mathematics teaching and learning (Voica et al., 2020). Likewise, PS is seen as an indispensable element of both mathematics and life (Kılıçkaya & Toptaş, 2017). Although PS has different meanings in different countries, it is seen as a main subject in both research and curricula worldwide (Törner et al., 2007). In addition, PS is the most important skill in demand and is seen as a vital element that helps students overcome future difficulties (Rahman, 2019).

The first necessary step in the examination of PS and PP skills is to examine the meaning of the concept of 'a problem' (Gonzales, 1998). Leavy and Hourigan (2021) reported that pre-service teachers (PSTs) have little experience and understanding of what a mathematically valuable problem is. Similarly, it was found that PSTs confuse the concepts of problem and practice (Yenilmez & Ev-Çimen, 2014). In contrast, Baş and Özturan-Sağırlı (2016), suggested comparing PSTs using examples related to the characteristics of the concept to create a correct perception of what is a problem. In addition, it is considered important to create a correct problem concept to improve PSTs' PS and PP skills to the desired level. Furthermore, Uygun et al. (2016) highlighted that it is important to understand both positive and negative perceptions about mathematical problems. Halmos (1980) also stated that at the heart of mathematics are problems, and he hoped that as teachers, we can train our students to be better problem-solvers and problem-posers than we are. Thus, it is important to investigate the meaning of the concept of a problem to create a correct perception of this concept, to understand the situations in both PS and PP, and to determine to what extent Halmos' hope has been realized.

It is obvious that these three concepts (problem, PP, and PS) are closely related and cannot be considered separately from each other. When students pose a problem, they must also think about its solution. This clearly shows the relationship between PS and PP (Chen et al., 2015; Silver, 1994). Kilpatrick (1987) suggested that the form and type of problems posed by students are good indicators for understanding how they will solve problems. It has been shown that even in simple PP studies changing the way that given problems are presented improved students' PS skills (Silver & Cai, 1996). It is also stated that the problems posed by students contain important clues about their mathematical abilities (Stoyanova, 1998). Some researchers found that students' success in PS was related to their PP skills, and therefore they used PP as a measure of PS success (Cai & Hwang, 2002; Cai et al., 2013; Silver & Cai, 1996). For example, Cai et al. (2013)

found that students who exhibit positive characteristics in PP are also strong problem solvers. However, although this strong relationship has been demonstrated, PS has a much more important place than PP (National Council of Teachers of Mathematics [NCTM], 2000). It is also noted that PP has not yet been included in mathematics curricula in ways that help students develop a deep and strong understanding of mathematics (Cai & Hwang, 2021). PP studies are still rare in both teacher education and classrooms (Bataller et al., 2022). It is stated that PP activities are not consistently or purposefully included in the mathematics curriculum (Cai & Jiang, 2017), and therefore teachers do not include PP activities in their lessons (Cai & Hwang, 2021). However, while the problems posed by a teacher shape the learning in their classroom, they help the teacher to understand the mathematical thinking of the students (Cai & Howson, 2013; Li et al., 2020; Silver, 1994). Being able to pose mathematically efficient problems is a critical element of high-quality mathematics teaching (Cai et al., 2015). For this reason, it is considered important to pose problems to teachers. Cai and Hwang (2021) state that PP should be an important part of mathematics curricula and practices due to its importance and potential. It is also stated that comparing students with PP activities is advocated in the mathematics curriculum of many countries (Li et al., 2022). It should be noted that PP has a fundamental position in the PS process (Leavy & Hourigan, 2021), PP activities develop PS and metacognitive skills (Akben, 2020), and that teaching mathematics with PP is like teaching mathematics with PS (Schroeder & Lester, 1989). PS and PP should therefore be considered together in mathematics. Since the problems, PS, and PP are intertwined, studies and activities related to these concepts should be included in curriculums, classroom environments, and teacher training programs together and frequently.

There is no significant evidence showing a negative effect of PP teaching and that it is a useful strategy to improve students' PS skills (Calabrese et al., 2022). One of the aims of mathematics teachers should be to educate students to have high PS and PP skills and who can increase their mathematical competence through these skills. Teachers are at the center of the implementation of educational innovations and developments (Cai & Hwang, 2020). There is potentially great value in helping teachers learn effective PP (Cai & Hwang, 2020) as teachers who understand PP will gain insight into their students' understanding of mathematics (Cai & Hwang, 2002; Kotsopoulos & Cordy, 2009). It is also suggested that more research is needed on what PP-based teaching entails in real practice (Zhang & Cai, 2021). It has been reported that in Turkiye PP studies inadequately contribute to mathematics education and that teachers and teacher training institutions do not have enough experience and knowledge about what PP is (Kanbur-Tekerek & Argün, 2019).

Despite all this evidence of the importance of PS and PP being considered together, recent studies have revealed that there is a lack of experience in PP among PSTs (Leavy & Hourigan, 2021). Their experience is generally limited to traditional PS, which focuses on arithmetic operations (Leavy & Hourigan, 2020), that PS is an important learning activity, but the quality of its teaching should be improved (Jiang et al., 2022), which is a strategy that is expected to be used by all students in their daily lives and future professions. Thus, the teaching of PS strategies is important so that students learn to manage their lives (Al Farra et al., 2022). Given all this evidence, it appears necessary for teachers to have acquired knowledge about both PS and PP while they are still PSTs. Testing and improving the abilities of PSTs, who have not yet started to work, is important in terms of both increasing the quality of mathematics lessons and understanding the measures to be taken in the training of teachers to be able to practice and teach these subjects effectively.

To sum up, it is important to understand the meaning of a problem correctly, comprehend both PS and PP processes, and recognize how these three concepts complement each other. Therefore, they should be considered together. Including studies and activities related to these concepts in curricula, classroom environments, and teacher training programs will help enhance students' skills while deepening their mathematical understanding. PP teaching is a valuable strategy for improving students' PS skills. Teachers play a crucial role in developing these skills and supporting them in learning effective PP, which can enhance their understanding of students' mathematical abilities. However, more research is needed on the practical application of PP, and in some areas like Turkiye, there is a lack of sufficient knowledge and experience in PP within mathematics education. Recent studies show that PSTs lack experience in PP, with their focus mostly on traditional PS, which is often limited to arithmetic operations. While PS is an essential learning activity, its teaching quality needs improvement. Teaching PS strategies is important for students to manage their lives and future professions. Therefore, it is crucial for teachers to gain knowledge of both PS and PP during their training, to improve the quality of mathematics lessons and better prepare teachers for effective practice and teaching.

Research Questions (RQ)

In the current study, the first aim was to reveal the perceptions about the concept of 'a problem'. The second aim was to investigate both posed and solved problems. To achieve these objectives, the following questions were tried answered:

RQ1. What are the perceptions of PSTs regarding the concept of problem?

RQ2. How are the PP-cases of PSTs?

RQ3. How are the PS-cases of PSTs?

Conceptual Framework

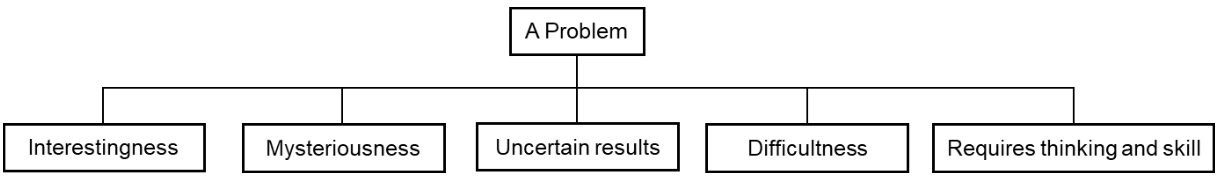
The Concept of a Problem

Since the existence of humanity, individuals have faced difficulties such as finding food, providing shelter, dealing with and communicating with wild animals, and so on, and struggled with all of them. However, this struggle is necessary for survival. In this context, a problem, which is expressed as something that is thrown in front of the individual and prevents him (Adair, 2000), is defined as a situation where the person wants to do something but does not know what to do (Altun, 2001). A problem is an event, activity, or subject for which the individual does not know a predetermined or memorized rule (Van de Walle, 2003). In addition, a problem is a situation that challenges the individual by shaking his self-belief at the first encounter and when the individual does not know the rule(s) for finding the solution (Katrancı, 2014). A problem has also been defined as a situation where an individual does not know how to reduce the distance between the place they are currently at and the place they want to reach (Rahman, 2019). Merriam and Tisdell (2016), on the other hand, describe a problem as a subject involving doubt, uncertainty, or difficulty. Thus, a problem may be defined as a difficulty that confuses the individual's mind and established thought processes but makes him want to solve it, but the individual does not know the method to find the solution.

Clearly, problems are related to real life. However, problems can also be related to mathematics. The problem of *'How many times higher is the invisible part of an iceberg than the visible part?'* is a real problem, while the problem of *'Can every even number be written as the sum of two prime numbers?'* is a mathematical problem (Altun, 2014). A problem requires more thought and skill before the right approach can be found, and a good problem is interesting and mysterious (Zeitz, 2007). It has been suggested that the result of a problem will be an uncertainty or difficulty, and that research, effort, and thinking are required for its solution (Yenilmez & Ev-Çimen, 2014). In summary, Figure 1 shows the characteristics of a problem.

Figure 1

The Characteristics of a Problem



Problems may be divided into two categories: routine and nonroutine. Routine problems are also known as ordinary problems. Altun (2014) expressed ordinary/routine problems as problems that are frequently encountered in daily life and require four operations. A problem that can be solved mechanically by an individual using past experiences in an almost similar or exactly the same situation is defined as a routine exercise (Nancarrow, 2004). For example, *'What is the average success rate of our class in mathematics?'* This problem is routine because we need to know it in our daily lives, and we can solve it mechanically with four operations. Nonroutine problems are also extraordinary problems. These types of problems require more thinking than routine problems. Mayer and Hegarty (1996) stated that a nonroutine problem arises when the individual who encounters it cannot immediately predict how to solve it. A nonroutine problem is defined as a task that is difficult to solve cognitively and that the solver does not know a method for solving (Selden et al., 2000). Such problems can be solved with the right strategies and different thinking skills (Kalaç & Çalışkan, 2022). For example, *'Hasan, the grocer,* *has a package of 2-, 3-, and 7-kilogram sugar and a scale. Due to Eid al-Fitr, he wants to prepare sugar packages from 1 to 9 kilograms without breaking the packages and using only them. Is it possible for Hasan to prepare the packages?'* The problem cannot be solved mechanically and is unusual because it requires thinking. In this context, nonroutine problems can be expressed as problems that the individual cannot solve immediately with the knowledge available on first encountering them.

Problem-Solving (PS)

Every problem needs to be solved whether it is routine or not. Merriam and Tisdell (2016) state that a person with a problem usually seeks a solution, an explanation, or a decision. This process is often referred to as PS, which is at the heart of mathematics (Cai & Hwang, 2021). PS is defined as eliminating the confusion in the mind of the individual (Silver, 1994), the individual's effort to reach a goal in the absence of an automatic solution (Schunk, 2012) and developing a solution or methods for resolving issues and questions (Karakuş-Aktan et al., 2021). In addition, PS is a planned process carried out to obtain solutions to both routine and nonroutine problems (Dostál, 2015; Goldhammer et al., 2014). PS, which includes an interaction between the experiences of the individual and the demands of the task, is not an advanced process reserved only for adult learners. Individuals of all ages should be able to be problem solvers (Martinez, 1998). There is a widespread consensus that teaching through PS encourages students' learning (Cai, 2003; Hembree & Marsh, 1993; Henningsen & Stein, 1997; Kroll & Miller, 1993; Schroeder & Lester, 1989; Stein et al., 1999). Cai (2003) states that students think about possible solutions while solving problems, then use one of these approaches and validate their ideas if they find a believable method of solution. Zhang and Cai (2021) also accept this situation as theoretically logical. On the other hand, Senemoğlu (2011) states that PS is an activity that requires choosing and using appropriate cognitive strategies with subject knowledge. It is also expressed as the elimination of the problem situation by using processing skills and existing knowledge through mental activities (Altun, 2008). Similarly, PS is expressed as a complex mental process that includes the use of learned information and the establishment of connections with abstraction, evaluation, and visualization (Gonzales, 1998). In summary, PS can be expressed as the process in which the confusion that occupies the mind is eliminated by using processing skills, as well as subject knowledge.

Performing the PS process, which is stated as a job that individuals of all ages should do, in a series of stages is recommended to facilitate coping with this complex process. These stages are i) understanding the problem, ii) planning, iii) implementing the plan, and iv) looking back (Polya, 1957; Senemoğlu, 2011). There are two basic questions that need to be answered in the first stage: understanding the problem. These are; i) What are the terms and conditions? and (ii) What is unknown/desired? Planning is the stage in which the relationship between what is given and what is desired is investigated. Implementing the plan involves solving the problem step by step using the strategy chosen for the solution. Although looking back is perceived by most individuals as checking the accuracy of the results, it has a broader meaning and is an enlightenment phase related to the PS process (Altun, 2008). In summary, these stages are i) understand, ii) plan, iii) solve, and iv) evaluate.

A fifth stage was added by Gonzales (1994). This stage is referred to as problem posing/creating. It was stated by Altun (2008) that another study on the evaluation of the solution to a problem is PP. In this context, the stages related to PS and PP are i) understanding the problem, ii) planning for the solution, iii) implementing the plan, iv) evaluating the solution, and v) posing the problem. However, if an individual is solving his own problem instead of a given problem, the step of understanding the problem can also be considered as a PP-step (Leung, 2009). When evaluated as a whole, these stages are understand-plan-solve-evaluate and pose.

Problem-Posing (PP)

PP, which is a special case of PS and is seen as the focal point of mathematics education (Christou et al., 2005; English, 1997; Kontorovich et al., 2012; Silver & Cai, 1996), is accepted as problem finding (Dickerson, 1999) and is a given involving reformulation of the problem (Mestre, 2002). In contrast, some researchers have defined PP as activities similar to PS but characteristically different (Baumanns & Rott, 2021; Dickman, 2014; Pelczer & Gamboa, 2009). Stoyanova and Ellerton (1996) see PP as a process. In this process, concrete situations are transformed into meaningful mathematical problems. Bonotto (2010) defines PP as the process of constructing personal interpretations of abstract situations by students and formulating them as meaningful mathematical problems, while Gonzales (1996) and Stickles (2006) describe this process as posing a new problem or reformulating an old problem. Unlike PS, in PP the focus is on generating problems based on situations (Cai et al., 2015). Silver (1994), on the other hand, defined PP as the rearrangement of a problem in the solution process or the creation of new problems to discover a mathematical situation.

Studies into generating new problems are expressed by Stoyanova and Ellerton (1996) under three headings, which are structured-PP (SPP), semi-structured-PP (SSPP), and free-PP (FPP). Special PS strategies are developed by teachers in SPP studies. What is expected from the students is to create problems that involve using these strategies in their solution. In SSPP studies, it is expected that problems will be created by students suitable for an open-ended situation. Visual figures, mathematical equations, or expressions can be used in such studies. In FPP studies, students are asked to pose problems without any limitations (Abu-Elwan, 1999; Christou et al., 2005; Dickerson, 1999; Stoyanova, 1998). All these activities have been reported to help increase the skills of analysis-synthesis and abstraction-generalization by developing creative and critical thinking, while also developing attitude, self-confidence, and motivation. It has been stated that these activities have positive effects in areas (Bayazit & Kırnap-Dönmez, 2017). Through PP, both students and teachers gain the opportunity to explore mathematical concepts (Silber & Cai, 2017).

The Place of PS and PP in the Turkish Educational System

In Turkiye, all teachers and middle (classes 5-8, and ages between 11 and 14) and primary (classes 1-4, ages between 7 and 10) school students follow the same curriculum, which is called 'mathematics curriculum (primary and middle school classes 1, 2, 3, 4, 5, 6, 7, and 8)', developed by the Ministry of National Education (MoNE, 2018). At the primary school level, there are four learning domains which are i) Numbers and Operations (NO), ii) Geometry, iii) Measurement, and iv) Data Processing (DP). There are 229 learning outcomes (of which 21 are related to PP and 31 are related to PS) related to these learning domains. Furthermore, there are 19 sublearning domains of which seven are related to NO, four are related to geometry, seven are related to measurement, and one is related to data processing. At the middle school level, there are 215 learning outcomes (of which 3 are related to PP and 33 are related to PS) associated with five learning domains, which are i) NO, ii) Algebra, iii) Geometry and Measurement (GM), iv) DP, and v) Probability. There are 39 sublearning domains, including 16 related to NO, five related to Algebra, 15 related to GM, two related to DP, and just one of them is related to Probability.

Method

The study was designed according to basic qualitative research principles. As words are used as data in qualitative research (Braun & Clarke, 2013), in this study, the meaning of the concept of problem was investigated based on the thoughts and statements of the participants. In addition, all qualitative research is concerned with how individuals make sense of their world and their lives and how meaning is constructed. The primary purpose of basic qualitative research is to reveal and interpret these meanings. For this purpose, data are collected through observations, interviews, or document analysis (Merriam & Tisdell, 2016). In the present study, the meaning of the concept of problem, PS, and PP cases were examined through the collected documents. The study sample consisted of 46 pre-service middle school mathematics teachers (PSM^{2}Ts) who were in their last year at a Faculty of Education, in a state university in Turkiye.

Data Acquisition Process

The data were collected with the 'Problem, Problem-Posing and -Solving Form (PPPSF)'. The PPPSF consists of two parts. The first part includes the question of *'What is a problem?'* to determine the perceptions of PSM^{2}Ts about the concept of problem. In the second part, the participants were asked to pose a problem at the middle school level and they should also provide the solution to their problem. Thus, the study was also an example of FPP activity. Since the study was not intended to evaluate the problems posed and solved in any mathematics subject, no explanation was given to the PSM^{2}Ts. The aim here was to determine what kind of problems they would pose and solve with their existing knowledge. Thus, it was hoped that it would be possible to see the general approach of PSM^{2}Ts in regard to PP and PS. Furthermore, by analyzing the study results, it may be possible to determine what improvements can be made in the evaluation of the posed problems. Similar studies can then be conducted in the context of specific mathematical topics.

The data collection tool was distributed to the PSM^{2}Ts, and they were informed that participation in the study was on a voluntary basis. They were given 50 minutes to fill out the form. This period is a course hour in education faculties in Turkiye. Time was considered sufficient for this study. It was also confirmed that the time was sufficient by asking the PSM^{2}Ts at the end of the study. They found the time sufficient. However, in cases where more than one problem was posed and solved, it would be possible to vary the duration. However, the time can be kept shorter if working with a group that frequently performs PP and PS practices. The duration could be considered a study variable, to observe the effects of applications. At the end of the period, the forms were collected back. In total, 48 data collection tools were obtained. These forms were coded as PST_{00}, PST_{01},…, PST_{47}.

Data Analysis

First, the 48 data collection tools were examined. Two data collection tools were not filled in and these were immediately excluded from the study. In addition, the first part of two forms, 'what is a problem?' was not completed so the data from 44 forms were used to answer RQ1. These data were evaluated by content analysis. In this analysis, the meanings and relations of certain words and concepts in the text or texts are determined, analysis is carried out, and inferences are made (Büyüköztürk et al., 2012). The data were coded, then the codes were brought together under certain themes, and the analysis was completed.

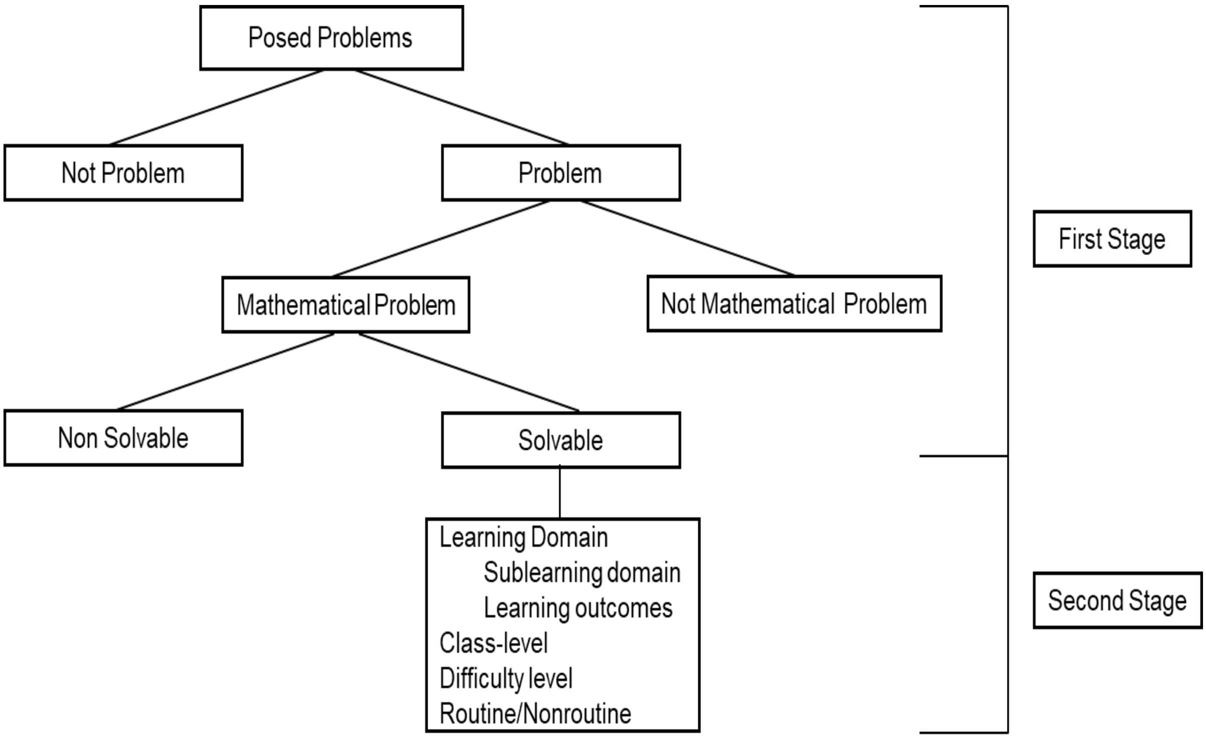
To answer RQ2, 46 forms were re-examined, and only 43 forms had problems, and the problems in these forms were evaluated. The literature review showed that researchers used certain criteria in the evaluation of the problems. For example, researchers (Silber & Cai, 2017; Silver & Cai, 1996) first determined non-problematic statements, non-mathematical problems, and mathematical problems in the classification of the problems, and then evaluated the problems according to whether they were solvable or not with certain complexity criteria. In another study, the criteria of quantity, originality, and complexity were discussed in the evaluation of the problems (Silver & Cai, 2005). Leung and Silver (1997) evaluated the problems posed in their studies according to whether they were mathematical problems, whether the problem was logical or not, whether the information provided was sufficient, and arithmetic confusion. They divided the problems posed in the first step of the evaluation, which they completed in three stages, into mathematical and nonmathematical. In the second step, they divided the mathematically determined problems into two categories: logical and non-logical. Finally, the problems determined as logical mathematical problems were evaluated according to whether the information given was sufficient or not. Korkmaz and Gür (2006) divided problems into five categories: open-ended, creative in terms of subject, four verbal processing problems, transforming expression, and impossible-tomeaningless. They evaluated them by assigning points to these categories, which were 25-15-10-10-0, respectively. Albayrak et al. (2006) evaluated the problems using a PP skill measurement tool that includes criteria such as i) not writing anything, ii) using all the given material, iii) adding new material, and iv) creating a sample from the given ones. Işık et al. (2011) evaluated the posed problems by the PSTs as not a problem but as an empty problem. The problems evaluated in the problem category were then divided into three: homework, relational, and conditional. Işık and Kar (2012a) initially divided the problems into two as appropriate and unsuitable for the data set, and then evaluated the problems suitable for the data set in terms of quantity and originality. Kılıç (2013), first, separated the problems in her study according to whether there were four operation problems or not, and then evaluated the four operation problems according to whether they were solvable or not. Kar and Işık (2015) divided the problems into two categories, which were problems and not problems, and divided the ones they identified as problems into two categories: valid and not valid. Xie and Masingila (2017) used the assessment rubric they adapted from the work of Rosli et al. (2013). Özdemir et al. (2022) first evaluated the problems as appropriate and inappropriate. After separating the problems identified as appropriate according to the task formats they determined, they were classified in the context of fluency, flexibility, and originality.

Therefore, in the present study, the problems were evaluated in two stages. In the first stage, the 43 problems posed were divided into two, according to whether they were a problem or not. Those evaluated as problems were then divided into two according to whether they were mathematical problems or not, and mathematical problems were evaluated according to whether they were solvable or not.

In the second stage, solvable mathematical problems were categorized into five: learning domain (Numbers and Operations (NO), Algebra, Geometry and Measurement (GM), Data Processing (DP), and Probability), sublearning domains, learning outcomes, class-level (5th-class, 6th-class, 7th-class, and 8th-class), difficulty level, and routine/nonroutine. Figure 2 shows the evaluation path of the posed problems.

Figure 2

The Evaluation Path: Evaluation Stages of Posed Problems



According to Figure 2, the cognitive domains of knowing, applying, and reasoning presented in the Trends in International Mathematics and Science Study [TIMSS] (MoNE, 2020) report were considered in determining the level of difficulty. The cognitive domain of knowing includes the information, concepts, and processes that students need to know. In this context, the posed problems at this cognitive domain level were determined as *'easy'*. The application cognitive domain focuses on applying knowledge to answer problems. Problems related to this area were determined as *'medium'*. The reasoning cognitive domain covers multistep problems and includes unusual situations. Kar and Işık (2015) revealed in their study that the operation structure was considered in evaluating the problems in the difficult category when posing problems related to different difficulty levels. In this framework, problems related to the cognitive domain of reasoning were interpreted as *'difficult'*. The point taken into consideration in determining whether the posed problem was routine or not was as follows. Baumanns and Rott (2021) state that whether a problem is routine or not varies from person to person. A problem that is not routine for an individual may be routine for someone who knows the appropriate solution. Accordingly, if the problem posed is of the type that any student studying at the relevant class-level can know and solve the relevant solution, it is classified as routine, if he knows the solution, it requires thinking and reasoning, and if it is more in number in the context of the process step, it is classified as nonroutine.

After the analyses were carried out (Figure 2), the evaluation was organized according to the expert opinion form. This form was sent to a mathematics teacher who has twelve years of experience in her profession and who is also continuing her doctoral education, and two experts in the field of mathematics education. They were asked to examine the evaluation and to state their opinions, if any. Opinion forms were examined, differences were re-evaluated, and the analysis was completed. In addition to this, the opinions of the experts on the problems were evaluated, and the findings obtained from these opinions are presented to the reader.

To attempt to find the answer to RQ3, the solutions provided for 42 problems were evaluated, using the rubric presented by Baki (2008), who suggested that problem solutions should be evaluated in five sections (see Table 3). These sections are i) Understanding the problem (UP), ii) Devising a plan (DP), iii) Implementing the plan (IP), iv) Evaluating (E), and v) Posing the problem (PP). Each section contains four criteria. The criteria are scored between 0 and 3 depending on whether the relevant skill is fully performed or not performed at all. Thus, the lowest score that PSTs can get from each section is zero and the highest score is three. The data were evaluated separately for each stage. For example, according to the key, the evaluation of the PST who scored 2+2+3+2+2=11 and the PST who got 3+2+2+2+2=11 have different meanings. This is because the first PST partially understood the solved problem, but the second PST understood the solved problem. Therefore, the differences between the two for different stages indicate differences in PS skills of the two PSTs despite achieving the same total score. Since the study determined the PS skills of PSTs, the scores for each section are given based on frequency (f) and percentage (%).

Validity and Reliability of Research

It is suggested that the concepts of persuasiveness and external validity should be used instead of the internal validity of qualitative research (Lincoln & Cuba, 1985; as cited in Erlandson et al., 1993). Credibility is about the consistency and clarity of research processes and results. In this context, the researcher needs to present evidence for the authenticity of the findings, the consistency of the process, and the objective collection of data (Yıldırım & Şimşek, 2016). Therefore, the research process and the mechanism by which the answers to each research question were found are explained in detail. In addition, it is considered important for validity to include direct quotations about the opinions of individuals interviewed in a study (Yıldırım & Şimşek, 2016). With this importance, to provide credibility and the criterion that is important for validity, both the problem and examples related to PP and PS are frequently included in the study. In addition, the differences in the fonts in these examples, the differences in the problem expressions, the problems and their solutions improve the credibility of the study. Transferability is about adequate description of the data, which can be achieved by detailed description. What needs to be considered in this is to organize the raw data according to the emerging concepts and themes and transfer it to the reader without comment (Yıldırım & Şimşek, 2016). Among the data obtained within the scope of this study, those related to the problem were presented without comment within the framework of certain themes. The data related to PP were presented by classifying them in the context of the concepts presented in the evaluation path, and no comments were made. Similarly, findings related to PS were presented in relation to the rubric.

Like validity, different concepts are used for the reliability of qualitative research. In this framework, the concepts of consistency instead of internal reliability and confirmability instead of external reliability are adopted (Lincoln & Cuba, 1985; as cited in Erlandson et al., 1993). Consistency should show itself in the development of data collection tools and analysis (Yıldırım & Şimşek, 2016). In the context of consistency, how the data of this study were obtained and how the analysis of the obtained data were carried out was presented in a clear and detailed way. In addition, the evaluation path and the conceptualization process of the data can be clearly observed. For this reason, I believe the study to be consistent. Confirmability is about staying away from assumptions and not adopting a subjective approach. However, additional methods such as peer confirmation and diversification are recommended to ensure confirmability (Yıldırım & Şimşek, 2016). As detailed in the data analysis, no subjective approach was adopted in the evaluations, no comments and no assumptions were made. Based on the evaluation path and rubric, the assessments were conducted openly. By considering the examples presented in the study, the processes made by different researchers and/or experts can be confirmed by testing. Likewise, confirmations of colleagues who are still mathematics teachers and continued their doctoral education and who are experts in mathematics education were also obtained. In this way, the external reliability of the study was ensured.

In addition, for the reliability of assessments made using rubrics, a single evaluator and inter-rater reliability should be confirmed (Moskal, 2000). In single evaluator reliability, the same evaluator repeats the assessment at different times (Moskal & Leydens, 2000). In inter-rater reliability, the same data should be evaluated by different evaluators (Newell et al., 2002). Therefore, problem solutions in the present study were evaluated by two different evaluators, and an evaluator repeated her evaluation three weeks later. Then, agreement between single and different raters was calculated using Cohen's Kappa statistics. The result of the evaluations of one evaluator was 0.96, and the agreement between two different evaluators was 0.90. If these values are between 0.81 and 1.00, this indicates perfect agreement between evaluators and between different time points (Landis & Koch, 1977). In summary, the present research was valid and reliable.

Findings

RQ1: What are the perceptions of PSTs regarding the concept of problem?

The aim here was to reveal the perceptions of PSM^{2}Ts about the concept of 'a problem'. Table 1 shows the findings concerning this concept.

Table 1

The Perceptions of PSM^{2}Ts toward the Concept of Problem

Themes	f	%
A case to be solved	25	56.82
An unpleasant situation	5	11.36
Difficulty	5	11.36
Trouble	5	11.36
Other	4	09.10

The perceptions of the PSM^{2}Ts about the concept of 'a problem' were grouped under five different themes. These themes are a case that needs to be solved, an unpleasant situation, difficulty, trouble, and other. The most common theme was a case that needs to be solved (56.82%, f=25). PSM^{2}Ts expressed the concept of problem as a situation, question, difficulty, set of procedures, and a mysterious forest. Figure 3 shows the answer of the PSM^{2}Ts coded PST$_{40}$.

Figure 3

The Response of the PST$_{40}$

'Çözülmesi gereken durumdur.'

'It is a situation that needs to be resolved.'

The PSM^{2}Ts also felt that a problem was an unpleasant, disturbing, and confusing situation. A problem was defined as any obstacle, difficulty encountered, and challenge to be overcome, according to the PSM^{2}Ts. Figure 4 shows PST$_{25}$'s explanation of the problem concept.

Figure 4

The Response of the PST$_{25}$

'Aşılması gereken zorluklardır.'

'The challenges that should be overcome.'

A problem has been defined as a problem whose solution is unknown and requires analysis and reasoning. Also, a problem has been seen as imitating a subject, mathematical operation complexity, proposition, and a whole or part of a phenomenon. Figure 5 shows the PST_{10}'s explanation of the problem concept.

Figure 5

The Response of the PST_{10}

'Belli bir sonucu olan matematiksel işlem karmaşıklığı'

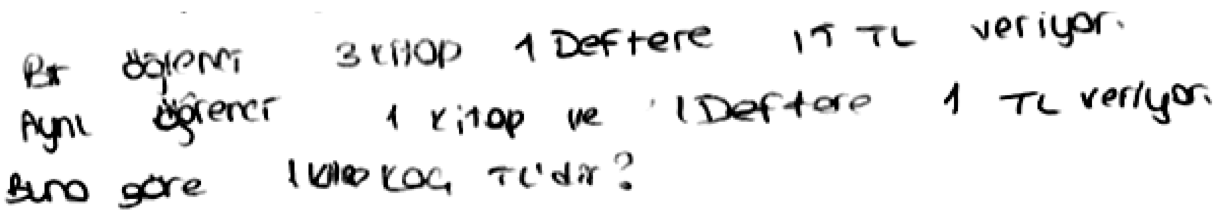
'A complexity of mathematical operations that have a certain consequence'

RQ2: How are the PP-cases of PSTs?

The first stage of evaluation of posed problems: All the problems in the 43 forms evaluated were identified as problems and mathematical problems. Then, these problems were analyzed according to whether they were solved or not. One problem was unsolvable. This problem is shown in Figure 6.

Figure 6

The Posed Problem by PST_{31} (Unsolvable)



'Bir öğrenci 3 kitap ve 1 deftere 15 TL veriyor. Aynı öğrenci 1 kitap ve 1 deftere 1 TL veriyor. Buna göre 1 kitap kaç TL' dir?'

'A student gives 15 TL for 3 books and 1 notebook. The same student gives 1 TL for 1 book and 1 notebook. Accordingly, how many TL is a book.'

When Figure 6 is examined, PST_{31} wants to find the price of a book. At first glance, the problem appears to be solvable. However, although the price of the book can be calculated as 7 TL with the information given, the price of the notebook is obtained as -6 TL. In this case, the problem is considered unsolvable.

The second stage of evaluation of posed problems: Moving on to the second stage of the analysis, the remaining 42 problems were evaluated in terms of learning domains, sublearning domains, learning outcomes, class-level, difficulty level, and routine/nonroutine. Table 2 shows the findings of the second stage of the analysis in detail.

Table 2

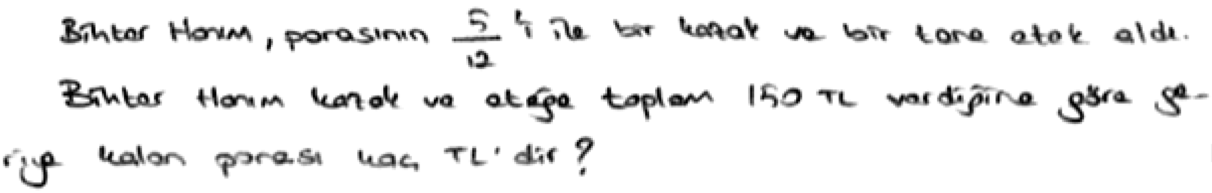
Second Stage Evaluation Findings of Posed Problems

PSM^{2}Ts' Codes	Learning Domain	Sublearning Domain	Subject	Class Level	Difficulty Level	Routine/Nonroutine
PST$_{00}$	NO	Operations with Fractions	Addition and Subtraction Problems in Fraction	5	Easy	Routine
PST$_{01}$	Algebra	Equation and Equation	Equation Problems	7	Middle	Routine
PST$_{02}$	GM	Geometric Bodies	Calculating the Volume of Prisms	6	Easy	Routine
PST$_{05}$	GM	Parity and Similarity	Similarity	8	Easy	Routine
PST$_{06}$	NO	Percentages	Percentage Problems	7	Easy	Routine
PST$_{08}$	GM	Measuring Length and Time	Length Measurement	5	Middle	Routine
PST$_{09}$	**NO**	**Natural Numbers**	**Operations with Natural Numbers**	**Primary school**	**Easy**	**Routine**
PST$_{10}$	Algebra	Equation and Equation	Equations	7	Easy	Routine
PST$_{11}$	NO	Square Root Expressions	Radical Numbers	8	Middle	Routine
PST$_{12}$	NO	Percentages	Percentages	7	Middle	Routine
PST$_{13}$	NO	Fractions	Finding the Complete Multiplicity Given a Fraction	5	Middle	Routine
PST$_{14}$	***Algebra***	***Linear Equations***	***Equations with First Order Unknown***	***8***	***Difficult***	***Nonroutine***
PST$_{15}$	Algebra	Equation and Equation	Equation Problems	7	Easy	Routine
PST$_{16}$	NO	Operations with Fractions	Fraction Problems	6	Easy	Routine
PST$_{17}$	NO	Decimal Notation	Problem Solving and Setting Up Decimal Notation	6	Very easy	Routine
PST$_{18}$	NO	Exponential Expressions	Exponential Numbers	8	Middle	Routine
PST$_{21}$	NO	Operations with Fractions	Fraction Problems	6	Middle	Routine
PST$_{22}$	NO	Operations with Natural Numbers	Natural Number Problems	5	Easy	Routine
PST$_{23}$	***Algebra***	***Linear Equations***	***Equations with First Order Unknown***	***8***	***Difficult***	***Nonroutine***
PST$_{24}$	NO	Integers	Integer Problems	6	Easy	Routine
PST$_{25}$	GM	Geometric Bodies	Calculating the Volume of Prisms	6	Easy	Routine
PST$_{26}$	NO	Operations with Natural Numbers	Natural Number Problems	5	Easy	Routine
PST$_{27}$	Algebra	Linear Equations	Equations with First Order Unknown	8	Easy	Routine
PST$_{28}$	NO	Operations with Fractions	Fraction Problems	5, 6	Medium, Easy	Routine
PST$_{29}$	Algebra	Linear Equations	Equations with First Order Unknown	8	Easy	Routine
PST$_{30}$	GM	Area Measurement	Calculating the Area of a Triangle Region	6	Middle	Routine
PST$_{32}$	NO	Fractions	Finding the Fraction of a Whole	5	Easy	Routine
PST$_{33}$	GM	Area Measurement	Area Land Problems	6	Middle	Routine
PST$_{34}$	NO	Percentages	Percentages	5	Middle	Routine
PST$_{35}$	NO	Operations with Fractions	Problem Solving and Setting Up Fractions	6	Middle	Routine
PST$_{36}$	NO	Operations with Fractions	Fraction Problems	5, 6	Medium, Easy	Routine
PST$_{37}$	NO	Percentages	Percentages	5	Middle	Routine
PST$_{38}$	**NO**	**Natural numbers**	**Operations with Natural Numbers**	**Primary school**	**Easy**	**Routine**
PST$_{39}$	NO	Operations with Fractions	Fraction Problems	6	Middle	Routine
PST$_{40}$	NO	Operations with Fractions	Fraction Problems	6	Middle	Routine
PST$_{41}$	NO	Operations with Fractions	Fraction Problems	6	Middle	Routine
PST$_{42}$	NO	Fractions	Finding the Fraction of a Whole	5	Easy	Routine
PST$_{43}$	***GM***	***Circle***	***Basic Concepts of the Circle***	***6***	***Difficult***	***Nonroutine***
PST$_{44}$	NO	Percentages	Percentages	6	Middle	Routine
PST$_{45}$	NO	Fractions	Finding the Complete Multiplicity Given a Fraction	5	Middle	Routine
PST$_{46}$	NO	Percentages	Percentages	5	Middle	Routine
PST$_{47}$	GM	Measuring Length and Time	Quadrilaterals and Their Properties	5	Easy	Routine

Findings regarding learning domains: PSM^{2}Ts were posed problems related to three different learning domains (NO, Algebra, GM) and 16 different sublearning domains. In total 27 (64.29%) problems were related to the NO-learning domain and these problems were related to nine different sublearning domains. The problems posed were for 12 different learning outcomes related to these sublearning domains. Figure 7 shows a problem in the NO-learning domain, fractions sublearning domain, and M.5.1.3.6. coded learning outcomes (MoNE, 2018).

Figure 7

The Posed Problem by PST_{13}

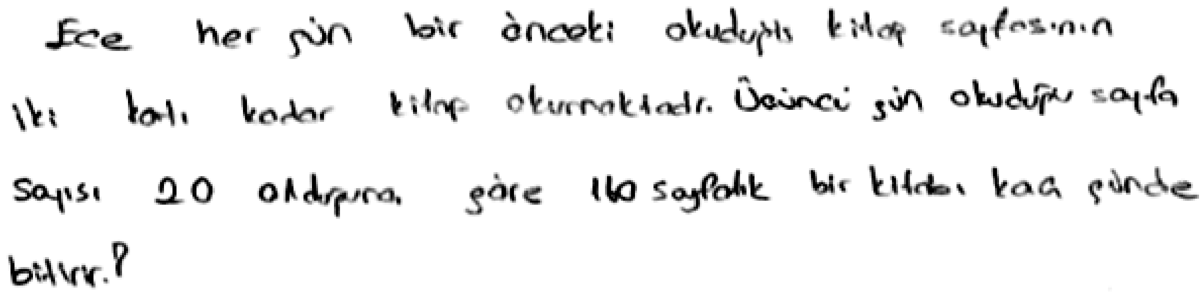

Bihter Hanım, parasının $\frac{5}{12}$'i ile bir kazak ve bir tane etek aldı. Bihter Hanım kazak ve eteğe toplam 150 TL verdiğine göre geriye kalan parası kaç TL'dir?

'Ms. Bihter bought a sweater and a skirt with 5/12 of her money. If Ms. Bihter gave a total of 150 TL for the sweater and skirt, how much is the rest of her money?'

In addition, seven (16.67%) problems were posed in the algebra learning domain. These problems were related to linear equations and equality and equation sublearning domains. The problems were set up for two different gains. In Figure 8, a problem related to the algebra-learning domain, equality and equation sublearning domain, and M.7.2.2.4. coded learning outcomes (MoNE, 2018) is given.

Figure 8

The Posed Problem by PST_{01}

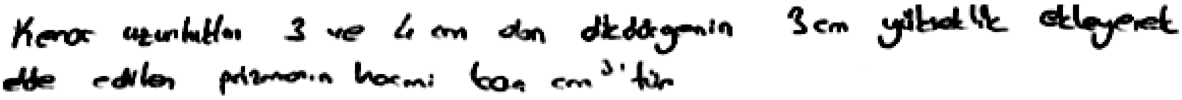

Ece her gün bir önceki okuduğu kitap sayfasının iki katı kadar kitap okumaktadır. Üçüncü gün okuduğu sayfa sayısı 20 olduğuna göre 160 sayfalık bir kitabı kaç günde bitirir?

'Ece reads twice as many books every day as the previous book page. If the number of pages she reads on the third day is 20, in how many days will she finish a 160-page book?'

Eight (19.04%) problems were posed in relation to the GM-learning domain. These problems are related to area measurement, circles, equality and similarity, geometric objects, and length and time measurement sublearning domains, and are established by considering seven different learning outcomes. Figure 9 shows a sample problem related to the GM-learning domain, geometric objects sublearning domain, and M.6.3.4.4. coded learning outcomes (MoNE, 2018).

Figure 9

The Posed Problem to PST$_{02}$



'What is the volume, in cm^3, of the prism obtained by adding 3 cm of height to a rectangle with side lengths of 3 and 4 cm?'

Findings regarding class-level: When the posed problems were evaluated according to the class-level, it became clear that two problems were at the primary school level. The first of these problems was in the NO-learning domain, the sublearning domain of multiplication with natural numbers, M.3.1.4.6. coded learning outcomes (MoNE, 2018) and is at the 3rd-class-level. The other problem was related to the NO-learning domain, multiplication with natural numbers sublearning domain, M.4.1.4.6. coded learning outcomes (MoNE, 2018) and was at the 4th-class-level. The problem related to the 3rd-class-level is presented in Figure 10 as an example.

Figure 10

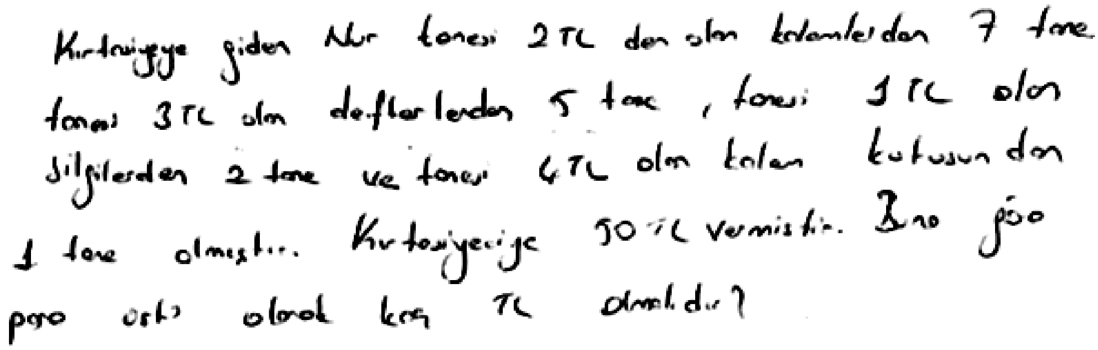
The Posed Problem by PST$_{09}$

'What is 5 times the number 44 minus 4?'

Fifteen of the remaining 40 problems were at the 5th-class-level, 12 at the 6th-class-level, six at the 7th-class-level, and seven at the 8th-class-level. The problems posed at the 5th-class-level were related to the learning domain of NO (f=13) and GM (f=2). At this level, in which five different sublearning domains (operations with natural numbers, fractions, operations with fractions, measuring length and time, percentages) are considered, a problem suitable for five different objectives was posed. In Figure 11, a problem posed at the 5th-class-level is presented.

Figure 11

The Posed Problem by PST$_{09}$



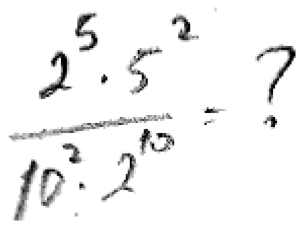
Kırtasiyeye giden Nur tanesi 2 TL den olan kalemlerden 7 tane tanesi 3 TL olan defterlerden 5 tane, tanesi 1 TL olan silgilerden 2 tane ve tanesi 4 TL olan kalem kutusundan 1 tane almıştır. Kırtasiyeciye 50 TL vermiştir. Buna göre para üstü olarak kaç TL almalıdır?

'Nur, who went to the stationery store, bought 7 pens with 2 TL, 5 notebooks with 3 TL, 2 erasers with 1 TL, and 1 from the pencil box, each of which was 4 TL. She gave 50 TL to stationery. Accordingly, how much TL should she receive as change?'

The problems posed at the 6th-class-level were also related to the learning domain of NO (f=7) and GM (f=5). The problems posed encompassed five different learning domains (area measurement, circle, geometric objects, operations with fractions, decimal notation) and are for seven different learning outcomes. Six problems were posed at the 7th-class-level and were related to the NO (f=3) and Algebra (f=3) learning domains. These problems were related to three different sublearning domains (equality and equation, operations with integers, percentages) and were posed for four different learning outcomes. Finally, the problems at the 8th-class-level were related to the learning domains of NO (f=2), Algebra (f=4), and GM (f=1). These problems encompassed four different sublearning domains (linear equations, parity and similarity, square root expressions, exponential expressions) and were related to four different learning outcomes. Figure 12 shows an example of a problem at the 8th-class-level.

Figure 12

The Posed Problem by PST$_{18}$

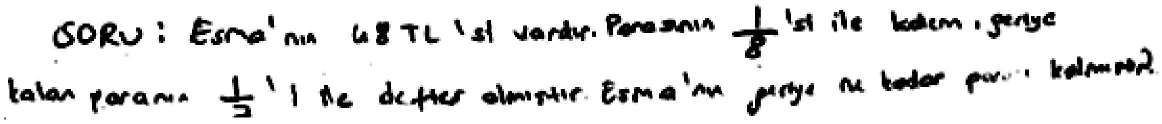


$$\frac{2^5.5^2}{10^2.2^{10}} = ?$$

Findings regarding difficulty level: Both problems posed at the 3rd and 4th-class-levels were, unsurprisingly, easy problems. When the remaining 40 problems were evaluated according to the degree of difficulty, three more problems were very easy, 16 problems were easy, 18 problems were moderate, and three problems were difficult. The problems that were considered very easy were related to the NO-learning domain and were at the 6th-class-level. Similarly, the problems that were evaluated as easy were related to each class-level. The problems with medium difficulty level were concentrated in the 5th and 6th-class-levels. Finally, the problems described as difficult are related to the learning domains of Algebra and GM at the 6th- and 8th-class levels. In Figure 13, an example problem posed at the 6th-class level of medium difficulty is presented.

Figure 13

The Posed Problem by PST_{36}

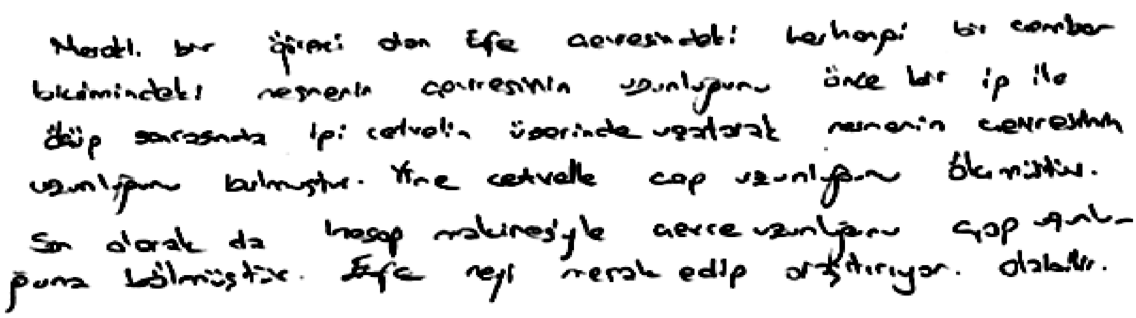


'Esma has 48 TL. She bought a pen with 1/8 of her money and a notebook with ½ of her remaining money. How much money does Esma have left?'

Findings regarding whether the problems routine or not: Only three of the posed problems were nonroutine problems that were related to Algebra and GM-learning domains, linear equations, and circle sublearning domains, and they were at the 6th- and 8th-class-levels. Figure 14 shows an example of a nonroutine problem at the 6th-class-level related to the GM-learning domain, circle sublearning domain, and M.6.3.3.2. coded learning outcome (MoNE, 2018).

Figure 14

The Posed Problem by PST_{43}

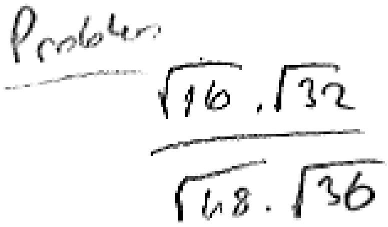


'Efe, a curious student, first measured the circumference of any circular object around him with a string and then stretched the string over the ruler to find the circumference of the object. He also measured the diameter length with a ruler. Finally, he divided the circumference by the diameter using a calculator. What could Efe be wondering and researching?'

Findings regarding experts opinions: The final evaluation regarding the problems posed concerned the opinions written by experts on the problems. The expert opinions were that there were errors in the expressions of the problems. For example, the statement *'The cake Ali bought, Ayşe ate 3/5 of it'* should be rewritten as *'Ayşe ate 3/5 of the cake Ali bought'*. Additionally, instead of phrasing the question as *'What is the result of the operation?'* with a question mark, it was observed that the question was directly stated. However, in some problems, punctuation marks (for example, punctuation marks such as question marks, commas) were not given importance, geometric figures were not drawn properly, and unnecessary information was included in geometry problems (for example, showing the similarity of triangles mathematically and showing it on the figure, etc.), and not a single question was asked at the root of the question (for example, … What is the initial number of items for Ayşe and what is the total number of items for Ali at the beginning?). In Figure 15, an example problem is given where neither the question stem nor any punctuation marks are included.

Figure 15

The Posed Problem by PST$_{11}$



$$\frac{\sqrt{16}.\sqrt{32}}{\sqrt{48}.\sqrt{36}}$$

When Figure 15 is examined, it is understood that it is desired to find the result, but *'equals question mark (= ?)'* to the given expression, *'What is the simplest form of the operation?'*, or *'What is the result of the operation?'* is missed and the question stem is not written in the form.

RQ3: How are the PS-cases of PSTs?

The solution of 42 problems was evaluated using rubrics. Table 3 shows the findings of RQ3.

Table 3

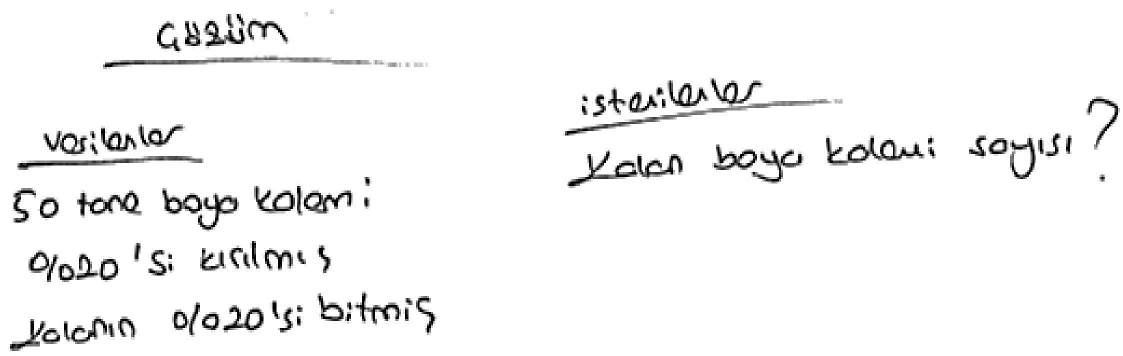
Findings Obtained from the Evaluation of the Solutions of the Posed Problems

	PS Criteria	f	%
UP	3: Fully understanding the problem	15	35.71
	2: Understanding a part of the problem	04	09.52
	1: Failure to understand the problem	00	00.00
	0: No effort is made to understand the problem	23	54.76
DP	3: Selecting a strategy that will lead to a suitable solution	00	00.00
	2: Selecting only one part of the strategy that will help the solution	00	00.00
	1: Choosing an inappropriate strategy	00	00.00
	0: No strategy selected	42	100.00
IP	3: Finding the correct and appropriate solution	39	92.86
	2: Making a solution that is partially correct	00	00.00
	1: Making a solution that is not appropriate and correct	00	00.00
	0: No solution can be made	03	07.14
E	3: Solving the problem and the new problem posed	00	00.00
	2: Logical verification of the results	01	02.38
	1: Partial validation of the results	00	00.00
	0: Not knowing how to verify the results	41	97.62
PP	3: The posed problem is logical and solvable	00	00.00
	2: A new problem was posed by changing the values of the problem	00	00.00
	1: There is a logical error in the posed problem and it cannot be solved	00	00.00
	0: Same problem or no problem written	42	100.00

According to Table 3, the problem was fully understood in the solution of 15 (35.71%) of the 42 problems, a part of the problem was understood in four (9.52%) problems, and no effort was made to understand the problem in the solution of 23 (54.76%) problems. Interestingly, no plan was prepared for the solution in any of the 42 (100%) problems, and no strategy was choosen. Figure 16 is an example of a well-understood problem solution.

Figure 16

Solution of PST$_{12}$


Çözüm

verilenler
50 tane boya kalemi
%20'si kırılmış
kalanın %20'si bitmiş

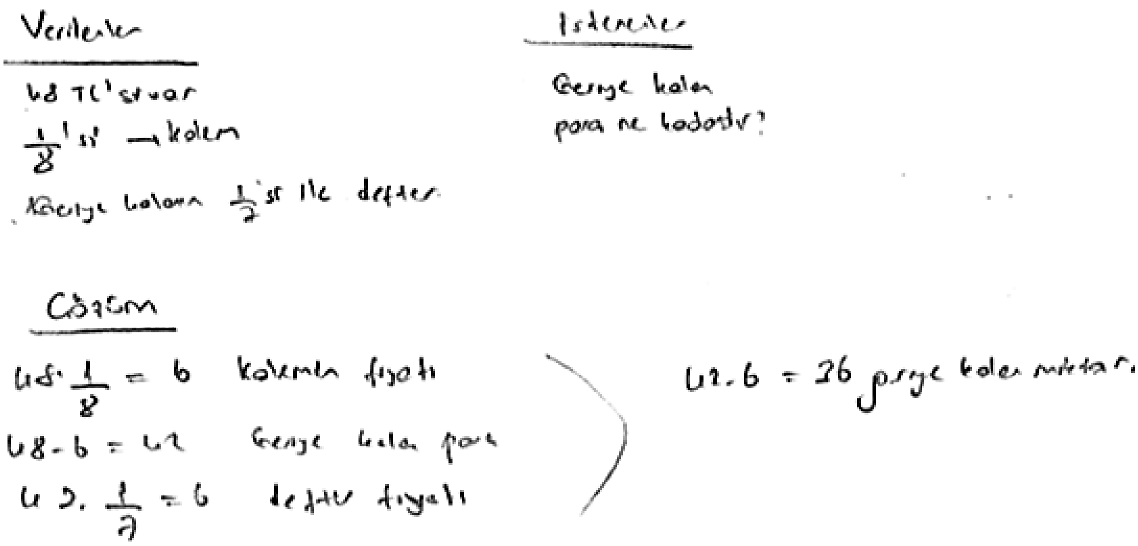
istenilenler
Kalan boya kalemi sayısı?

Given	Wanted
50 crayons	The number of crayons remaining?
20% broken	
20% finished	

Three (7.14%) PSTs did not solve the problem they were posed, while the other PSTs (92.86%) reached the appropriate and correct solution. One (2.38%) PST made a logical evaluation of the solution, nothing was done to evaluate the results obtained in 41 (97.62%) PS. Regarding the PP stage given in Table 3, neither the same problem nor any other problem was written after 42 (100%) solutions. An example problem solution is presented in Figure 17, in which a fully understood and appropriate solution is obtained.

Figure 17

The Complete Solution of PST_{36}



Given

He has 48TL

1/8 pencil

Notebook with ½ of the remainder

Wanted

How much money is left?

Solution

48.1/8= the price of 6 pens

48-6= 42 remaining money

42.1/2= 6 notebook prices

42.6= 36 remaining amount

When Figure 17 is examined, it is seen that PST_{36} fully understood the problem, did not determine any strategy for the solution, did not plan, proceeded to the solution after understanding the problem, and did not make any evaluation after reaching the correct solution.

Conclusion and Discussion

The first aim of the study was to determine how the concept of ‘a problem’ is perceived by the PSM^2Ts. The general findings were that a problem is a difficulty that the individual does not like the moment s/he encounters it, requires analysis and reasoning, but needs to be solved. Thus, they had gained information about the meaning of the concept of a problem within the scope of undergraduate courses. In this context, I expected that the result obtained should be meaningful. However, it may be that the reason why they attribute these expressions to the concept of problem, which they often hear and encounter in their lives, is that the concept of problem is expressed as the question, issue, and problem to be solved and not just mathematics problems. In addition, the example of 'problem child' points to those whose behavior is not normal (Turkish Language Institution [TLI], 2025). PSM^{2}Ts often encounter such examples in their daily lives. This may be among the reasons for the meaning they attribute to the concept of problem. Baş and Özturan-Sağırlı (2016) also concluded that the concept of 'problem' is defined as a problematic situation. In addition, they concluded that the words 'need to be solved' and 'difficulty' are the most used words according to the phrases used by the candidates while describing the problem. Similarly, Uygun et al. (2016) reported that both mathematics and primary school teacher candidates attributed the expressions difficult and complex to mathematical problems. Kaba (2018) also found that the concept of problem was associated with the category of difficulty. As a result, the need to solve a problem and its expression as a difficulty are common points in these studies. The reason for this is that a problem situation causes distress to individuals. Turhan-Türkkan and Yeşilpınar-Uyar (2016) showed that the category of 'mathematical problem as a difficult and complex activity' came to the fore, while Sezgin-Memnun (2015) also found that a mathematical problem was perceived as a difficult and complex concept. These results were obtained in studies conducted with middle school students. In this case, it is seen that both future teachers and students have common perceptions of a mathematical problem and describe a problem as difficult and complex. The reason for this may be that the existing thoughts and attitudes of those who do the teaching are transferred to their students. Therefore, positive thoughts of teachers in regard of mathematical problems and knowledge of the correct definition of a concept are important. Likewise, Yenilmez and Ev-Çimen (2014) concluded that PSTs confuse the concepts of problem and exercise, and that the number of candidates who write exercises instead of problems is high. In this context, it is important that the candidates who teach mathematics should know the meaning of the concept of problem correctly. It is important to create a correct perception about the concept of problem to improve PSTs' skills using PP and mathematical problems to the desired level (Baş & Özturan-Sağırlı, 2016). In summary the perceptions of PSM^{2}Ts regarding the concept of 'problem' suggest that they generally define problems as difficult situations that need to be solved. Both PSTs and students may often perceive mathematical problems as complex and difficult. This perception could potentially be shaped by teachers' attitudes toward mathematical problems, which they may pass on to their students. Additionally, PSTs may confuse problems with exercises, which could lead to conceptual confusion, emphasizing the importance of learning the correct definition of a problem.

On examination, the posed problems were mathematical and solvable, except for one. Thus, it seems that most PSM^{2}Ts can pose mathematical problems with solutions. Kaba (2018) also reported that the problems posed in her study were mathematical, as did Kar and Erkan (2022) but these authors also reported that PP tendencies changed according to the problem type. Kılıç (2017) concluded that more than half (55%) of the candidates could successfully pose a problem. Bataller et al. (2022) and Şengül and Katrancı (2014; 2015) also found that future teachers pose solvable mathematical problems, likely due to the effects of the education they have received. Erkan and Kar (2022) concluded that most of the candidates started the PP process by finding a solution, suggesting that the solutions to the posed problems were also taken into consideration. Another reason for the result may be related to the metacognitive skills of PSTs. Metacognitive skill allows for assessment of whether a problem is well defined and solvable (Baumanns & Rott, 2023). My findings, and those of other researchers, suggest that the PSM^{2}Ts use their metacognitive skills well.

The problems provided related to only three different learning domains and there were no problems related to the DP and Probability-learning domains; the problems mostly related to the NO-learning domain. Similarly, Kaba (2018) also reported that most problems were related to the NO-learning domain. This may occur because the achievements related to the NO-learning domain are more related to PP and that PP studies for this learning domain are frequently carried out. Işık and Kar (2012b) found that teachers posed problems that excluded the geometry domain. As both pre-service and inservice teachers ignore some learning domains, there may be an inadequacy of content information related to the learning domains avoided which should be addressed in teacher training.

I found that mostly of the posed problems were at the 5th-class-level, and fewer problems were posed at higher classes. No problem was posed regarding the probability-learning domain, which is taught only in the 8th-class. Kaba (2018) reported that most problems posed by her cohort were at the level of the 6th-class. When the mathematics curriculum in Turkiye is examined, M.5.1.4.2 coded learning outcomes at the 5th-class-level as well as M.5.1.2.12. and M.6.1.1.4 coded learning outcomes at the 6th-class-level are related to PP (MoNE, 2018). These results are of importance because, despite indicating that the education given in faculties of education progresses in harmony with the curriculum, being able to pose problems for the 7th- and 8th-class-levels is important because of the significant relationship between PP and PS skills (Kar et al., 2010).

In the present study the posed problems were generally either easy or medium difficulty. Similarly, Özgen (2019) also evaluated the difficulty of posed problems and found that most tended to be of medium difficulty. Şengül and Katrancı (2014) determined that 80.95% of the candidates posed problems such as exercises and 14.05% posed simple problems. The same researchers (2015) concluded in their other study that 69% of the candidates wrote practice-type problems. Bayazit and Kırnap-Dönmez (2017) evaluated the problems in their studies according to three levels of difficulty: knowledge, practice, and reasoning. Of these, 10.3% were of the knowledge type, 67.6% were practice, and 22.1% were reasoning. Again, candidates tended to pose problems of medium difficulty. Peng et al. (2022) also found a similar result as teachers in their study tended to pose problems of low or medium difficulty. Sappaile and Djam'an (2017) suggested that teachers should perform exercises to pose problems that are neither difficult nor easy, encouraging students to think creatively. In these studies, it is observed that problem posers generally prefer easy or medium-difficulty problems. This preference could be based on the idea that medium-difficulty problems are more suitable for enhancing students' success and encouraging them to think. Additionally, educators choose problems that match students' current knowledge and skills, ensuring they are neither too difficult nor too easy, thereby supporting the PS process. This trend provides opportunities for students to develop their creative thinking skills while minimizing the risk of decreasing motivation, which can occur with problems that are too hard or too easy.

About 93% of the posed problems were routine and thus PSM^{2}Ts in my cohort were capable of posing routine problems. Baş and Özturan-Sağırlı (2016) also concluded that candidates pose routine problems with mathematical content. Similar findings were also published by Kaba (2018) and Bayazit and Kırnap-Dönmez (2017) who added that problems with a routine character can be solved by applying certain algorithms. These findings, taken collectively, suggest that PSTs do not go beyond the routine. This may be because the PSTs do not encounter enough PP activities. Crespo and Sinclair (2008) stated that PSTs rarely encounter the opportunity to pose their own problems, and they solve the problems that teachers pose or that are in the textbooks. However, experience is very important in PP, and it has been shown that those who have participated in PP studies are more successful than those who did not (Güveli, 2015). Or and Bal (2023) also concluded that students are more successful in solving routine problems and stated that more nonroutine problems should be included in mathematics classes and books. That the candidates from the present study mostly posed routine problems may be because such problems are mostly included in mathematics books. PP has been reported to have a marked effect on mathematical creativity (Hendrajaya et al., 2022) and is important for the posing of nonroutine problems. Thus, the more PP activities that are faced, the more mathematical creativity will increase, so that mathematics teachers will be able to deal with, and teach, more than simply routine problems.

In terms of PP, the final findings are to do with the construction of the actual problem. These included problems with the text of the problems, not paying attention to punctuation marks, not drawing geometric figures properly, giving unnecessary information, and not asking a single question in the root of the question. Barham (2020) also concluded that mathematical terminology was used in a limited or incorrect fashion. Kar and Işık (2015) reported that a teacher who participated in their studies could not form the root of the question. The poor use of mathematical terminology and the inability to create problem texts appear to be common deficiencies, perhaps because of deficiencies in mathematics concept-content knowledge and reading skills. Likewise, not paying attention to punctuation marks may be related to inadequate reading habits.

More than half of the PSTs (54.76%) did not make any effort to understand the problem in PS. This finding is concerning as the text-based nature of the problems requires reaching a conclusion after reading and understanding, and it has been shown that those who have problems with reading comprehension will also have difficulties in PS (Karakuş-Aktan et al., 2021). Proceeding to a solution without understanding the given problem may result in giving a wrong answer. Topbaş-Tat (2018) found that teacher candidates did not pay any attention to these steps before being taught about PS, but subsequently the candidates both paid attention and realized the importance of understanding the problem.

No strategy for problem solution and no preparation for the solution process were presented by any of the PSM^{2}Ts. This may be because mathematics teachers are result-oriented while solving problems (Gürbüz & Güder, 2016). Alternatively, they may not have been taught about the stages of PS throughout their education life or have forgotten them. However, PS strategies have been shown to be extremely important, accounting for 80% of PS success (Altun & Sezgin-Memnun, 2008). Unfortunately, mathematics teachers also have deficiencies in developing varying solutions to a single problem (Gürbüz & Güder, 2016). Çeker and Ev-Çimen (2017) reported that teachers use unplanned PS strategies without being aware of them. Teachers at the beginning of their profession used strategies remembered from undergraduate education, while senior teachers used strategies acquired through experience. Avcu and Avcu (2010) also concluded that strategies were used but were limited in terms of diversity. Altun et al. (2007) found that almost all questions were attempted using the single strategy of equation building, which may be due to the uniformity in our education system. These findings, taken together, highlight the importance of undergraduate education and that training about the stages of PS should be given. Interestingly, Guzman-Gurat (2018) reported that PSTs did use various PS strategies, perhaps because of differences in the study populations used. Most of the evidence points to deficiencies in the training of mathematics teachers, with Yayuk and Husamah (2020) showing that teacher training activities do not encourage students to plan while determining PS strategy. However, strategies should be used to implement the PS stages (Avcu & Avcu, 2010).

While logical verification of the result was presented for only one problem, evaluation of the validity of the result obtained in any of the other problems was not performed. PP studies include evaluation of the result obtained using a different strategy and this clearly did not happen in my cohort. Yılmaz (2019) found that just over a third (36%) of PSTs evaluated solutions using a different strategy. This proportion was much lower (8%) in the study of Yayuk and Husamah (2020), and disappointingly was only 2.3% in my study. The reason may be self-belief or self-confidence that they will not make mistakes. However, Şengül and Katrancı (2012) showed that candidates do not pay attention to or ignore the PS stages, and this may also have occurred in this study.

Recommendations

The activities can be organized to help PSTs correctly distinguish between concepts like ‘problem’ and ‘exercise’. In these trainings, the differences between problems and exercises should be highlighted, and the role of each concept in mathematics teaching should be explained. It is the classroom teachers who will lay the foundation of mathematics for the future general population. Thus, as the heart of mathematics is the idea of a problem, PS and PP, it is important for them to know the correct definition of ‘a problem’ and techniques for PS and PP. To achieve this, teachers should receive indepth training on these topics. Teacher education programs should not only cover PS and PP techniques but also provide practical examples of how these concepts can be applied in the classroom. Additionally, teachers should engage in hands-on exercises to develop these skills, which will help foster their ability to encourage creative PP in students. This type of training will enable teachers to lead more effective lessons and support students in developing their mathematical thinking skills.

It is noteworthy that no problems were posed in the learning domains of probability and DP. This results in an inadequate representation of these domains in mathematics instruction. Geçici and Türnüklü (2020) also concluded in their thematic analysis that only a few theses were written related to the DP learning domain and suggested that PP studies related to this field be conducted. I found that most of the problems were related to the NO-learning domain. The reasons for this bias should be investigated further, possibly through conducting interviews with PSM^{2}Ts. I also suggest that there is a need for activities and studies into PP related to other learning domains. My other suggestion is to diversify PP activities in teacher training programs to ensure that PSTs gain sufficient knowledge and experience across all learning domains. In particular, more problems related to DP and probability should be incorporated to address the existing content gaps in these areas. By doing so, teachers will be better equipped to adopt a balanced approach to teaching and provide a more comprehensive mathematical education.

If teachers are only setting easy, medium difficulty, and routine problems, then the teaching of mathematics will be similarly limited. PP is a process of higher-level mental actions that includes PS, and metacognition should be employed for success (Bayazit & Kırnap-Dönmez, 2017). Thus, I recommend conducting more difficult and nonroutine PP studies to provide a broader range of success assessment, prepare students for high-level thinking, and develop their logical reasoning skills. Education faculties need to increase focus on this issue so that PSTs are more competent when starting their careers. Moreover, nonroutine problem examples should be given more place in mathematics textbooks, and both teachers and students should be faced with such question solutions.

Teacher education should also stress the teaching of PS strategies, considering the evident effect of PS on success. At this point, it is recommended to examine all levels, since the contribution of PS strategies to PS success is at different levels. This recommendation was made almost 20 years ago but the problem has persisted to the present, at all levels of the teaching of mathematics (Altun et al., 2007).

Some of the PSTs were unable to correctly draw geometric shapes when designing their problem. I suggest that PSTs should encounter dynamic geometry software more often during their education. Similarly, there are smart boards in our schools. Using them should enable the creation of neat drawings of related figures and objects. One of the reasons why the PSTs were unable to draw correctly may have been that they are used to using software or smart boards to help them and so drawing free hand was challenging. I conjecture that deficiencies in the text parts of the posed problems may be related to PSTs reading habits, it is recommended to conduct a study to test this idea. For example, PP and PS skills can be examined according to the frequency and type of book read and the text produced by readers could be compared to the text produced by non-readers.

Limitations

The posed problems were analyzed according to the evaluation path which was based on the existing literature. However, expert opinion was that the problems should also be evaluated linguistically. Although these findings are included in the study, the evaluation path used can be seen as limited in this aspect. To eliminate this limitation, it is recommended to conduct analyses according to different evaluation paths. The study was limited to 46 PSM^{2}Ts. Since it is a qualitative study, this number was sufficient, but it would be beneficial to repeat the study with larger samples. Thus, for example, it is possible to see whether there are any problems related to learning domains where no problems are posed. The research can be seen as limited in the context of the rubric used to evaluate the solutions. Use of different rubrics may have produced different results so the optimal rubric for assessment should be investigated.

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Genişletilmiş Türkçe Özet

Matematik öğretmenleri, matematiğin temellerini atacak kişilerdir. Bu nedenle, matematiğin kalbinin problem, problem çözme ve kurma fikri olduğunu göz önünde bulundurursak, öğretmenlerin problemin doğru tanımını ve problem çözme ile kurma tekniklerini bilmeleri önemlidir. Öğretmenlerin hem problem çözme hem de problem kurma konularında bilgi sahibi olmaları, öğretmen adayları iken kazanmaları gereken bir durumdur. Henüz mesleğe başlamamış olan adayların yeteneklerini test etmek ve geliştirmek, matematik derslerinin kalitesini artırmak ve bu konuları etkili bir şekilde uygulayabilmek ve öğretmek için öğretmen yetiştirme sürecinde alınması gereken önlemleri anlamak açısından önemlidir. Bu sebeple bu çalışmanın birinci amacı, 'problem' kavramına yönelik algıları ortaya koymaktır. İkinci amaç ise kurulan ve çözülen problemleri incelemektir. Bu hedeflere ulaşmak için aşağıdaki araştırma soruları (AS) cevaplanmaya çalışılmıştır:

AS1. Öğretmen adaylarının problem kavramına yönelik algıları nelerdir?

AS2. Öğretmen adaylarının problem kurma durumları nasıldır?

AS3. Öğretmen adaylarının problem çözme durumları nasıldır?

Çalışma, temel nitel araştırma ilkelerine göre tasarlanmıştır. Nitel araştırmalarda veriler olarak kelimeler kullanıldığından (Braun & Clarke, 2013), bu çalışmada problem kavramının anlamı, katılımcıların düşünceleri ve ifadeleri üzerinden incelenmiştir. Ayrıca, tüm nitel araştırmalar bireylerin dünyalarını ve yaşamlarını nasıl anlamlandırdıklarına ve anlamın nasıl inşa edildiğine odaklanır. Temel nitel araştırmanın birincil amacı, bu anlamları ortaya koymak ve yorumlamaktır. Bu amaç doğrultusunda veriler; gözlemler, görüşmeler veya doküman analizi yoluyla toplanır (Merriam & Tisdell, 2016). Bu çalışmada, problem kavramının anlamı, problem çözme ve kurma durumları, toplanan belgeler aracılığıyla incelenmiştir. Çalışma grubu, Türkiye'deki bir devlet üniversitesinin Eğitim Fakültesinde son sınıf öğrencisi olan 46 ortaokul matematik öğretmeni adayından oluşmaktadır. Veriler, 'Problem, Problem Kurma ve -Çözme Formu (PPKÇF)' ile toplanmıştır. PPKÇF iki bölümden oluşmaktadır. İlk bölümde, öğretmen adaylarının problem kavramına yönelik algılarını belirlemek amacıyla 'Problem nedir?' sorusu yer almaktadır. İkinci bölümde ise katılımcılardan ortaokul seviyesinde bir problem kurmaları ve kurdukları bu probleme çözüm sunmaları istenmiştir. 'Problem' kavramına ilişkin veriler, içerik analizi yöntemiyle değerlendirilmiştir. Kurulan problemler, bir değerlendirme yol haritası kullanılarak analiz edilmiştir. Çözülen problemler ise bir rubrik kullanılarak değerlendirilmiştir.

Sonuçlar, 'problem' kavramının, bireyin karşılaştığında hoşlanmadığı, analiz ve mantık gerektiren ve ayrıca çözülmesi gereken bir soru olarak algılandığını göstermiştir. Kurulan problemler matematiksel ve çözülebilir niteliktedir. Veri işleme ve olasılık öğrenme alanlarına ilişkin problem kurulmamıştır. Genellikle beşinci sınıf seviyesinde, kolay ve orta zorlukta problemler oluşturulmuştur. Problemlerin neredeyse tamamı (%93) rutin problemlerdir. Sonuç olarak, öğretmen adayları çözülebilir rutin matematiksel problemler kurmuşlardır. Benzer şekilde, bu problemlerde matematiksel terminoloji kullanımı ve problem metni oluşturma konusunda eksiklikler bulunmaktadır. Problem çözme sürecinde ise, öğretmen adaylarının yarısından fazlası problemi anlamaya odaklanmamış, hiçbir problem çözmede stratejisi seçilmemiş ve yalnızca problemlerin kısmi mantıklı değerlendirmeleri yapılmıştır. Özetle, problem çözme aşamaları dikkate alınmadan sonuca yönelik çözümler yaygın bir şekilde görülmüştür.

Kurulan problemler, mevcut literatüre dayalı bir değerlendirme yol haritasına göre analiz edilmiştir. Ancak, uzman görüşleri, problemlerin dilsel açıdan da değerlendirilmesi gerektiği yönündedir. Bu bulgular çalışmada yer almış olsa da kullanılan değerlendirme yol haritası açısından bu çalışma sınırlı olarak görülebilir. Bu sınırlılığı ortadan kaldırmak için farklı değerlendirme yol haritaları kullanılarak benzer yapıda bir çalışmanın yürütülmesi önerilmektedir. Çalışma, 46 öğretmen adayı ile sınırlıdır. Nitel bir çalışma olduğu için bu sayı yeterli olmuş olsa da daha geniş örneklemlerle çalışmanın tekrarlanması faydalı olacaktır. Böylece, örneğin, hiç problem oluşturulmayan öğrenme alanlarına ilişkin herhangi bir problem yazılıp yazılmayacağı görülebilir. Araştırma, çözümleri değerlendirmek için kullanılan rubrik açısından da sınırlı olarak değerlendirilebilir. Farklı rubriklerin kullanılması farklı sonuçlar doğurabilir, bu nedenle değerlendirme için farklı rubriklerin kullanılması önerilmektedir.

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